

Are superheavy stable quark clusters viable candidates for the dark matter?

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. The explanation for the origin of families of quarks and leptons and their properties is one of the most promising ways to understand the assumptions of the *Standard Model*. The *Spin-Charge-Family* theory [1–12], which does propose the mechanism for the appearance of families and offers an explanation for all the assumptions of the *Standard Model*, predicts two decoupled groups of four families. The lightest of the upper four families has stable members, which are correspondingly candidates to constitute the dark matter [9]. We study the weak and the "nuclear" (determined by the colour interaction among the heavy fifth family quarks) scattering of such a very heavy baryon by ordinary nucleons in order to show that the cross-section is very small and consistent with the observation in most experiments so far, provided that the quark mass of this baryon is about 100 TeV or above.

I. INTRODUCTION

The *Standard Model* has no explanation for the existence of families and their properties as well as for the appearance of the scalar field, which in the *Standard Model* determines the properties of the electroweak gauge fields and, together with the Yukawa couplings, the properties of families of quarks and leptons. A theory which would explain the origin of families and the mechanism causing the observed properties of quarks, leptons and gauge fields is needed.

The *Spin-Charge-Family* theory [1–12], which offers the explanation for all the assumptions of the *Standard Model*, with the appearance of families, of the Higgs's scalar and the Yukawa couplings included, is very promising for this purpose. The theory predicts that there are the stable fifth family baryons, which constitute the dark matter [9]. The main features of this theory are summarized in Sect. II.

The properties of the fifth family members, their freezing out in the evolution of the universe and also their interaction with the ordinary matter were evaluated in Ref. [9]. In this paper we study scattering of these stable fifth family baryons on the ordinary nucleons, when this very small object of the mass about one hundred TeV approaches with the velocity of our Sun an ordinary nucleus. The nucleus, which is six orders of magnitude larger and three orders of magnitude lighter than

the very heavy baryon, recoils elastically from the very heavy baryon, as long as is the excitation energy of the ordinary nucleus much smaller than the kinetic energy of the relative motion of both objects. We test whether the results agree with the present experiments [13–19].

In this paper we present:

- i. We show under which conditions the electrically neutral baryon $n_5 = (u_5 d_5 d_5)$ is the lightest [9] (the subscript 5 is used to denote the fifth-family members). One might expect that a charged combination $(u_5 u_5 u_5)$ or $(d_5 d_5 d_5)$ would be the lightest, depending on whether u_5 or d_5 is lighter (what depends on the scalar and other interactions, which determine masses of the stable fifth family members). We present limits on the u-d mass difference such that the neutral combination is in fact the lightest.
- ii. Since the quark clusters always carry the weak charge, we present the constraint that the corresponding weak scattering does not exceed the upper bound estimated in present observations. It turns out that in the (light cluster) - (very heavy cluster) scattering the weak cross section limits to a value around 10^{-13}fm^2 . This means that the low rate of collisions of the fifth family candidate for the dark matter with the ordinary matter requires a low number density of dark matter particles to be in agreement with the observations. From the dark matter mass density [20] it follows that the fifth family baryon mass must be high - around 100 TeV or above.
- iii. To binding of the ordinary quarks in nucleon many gluons contribute. The gluon field around the quarks makes the main contribution to the gluon mass, manifesting in the potential which grows with the distance between two quarks. When the ordinary nucleon (of the size of fm) scatters on another ordinary nucleon, the nuclear force always dominates over the weak force. Very heavy quarks are bound, due to the heavy mass, mostly through one gluon exchange force. When a light ordinary nucleus scatter on a colour neutral very heavy baryon, a first family quark of the nucleon sees the colour force of the heavy baryon only due to the induced colour dipole - colour dipole, and higher multipole, interaction. We explore smallness of the colour dipole - colour dipole cross section in dependence of the mass of the fifth family quarks. We show that the very heavy quark masses, which satisfy the bound for weak scattering, also satisfy the bound for strong scattering.

This paper follows to some extent the Ref. [22–24].

II. THE SPIN-CHARGE-FAMILY THEORY

In this section a short introduction to the *Spin-Charge-Family* theory [1–10, 10–12] is presented. Only the essential things are reviewed. The reader can skip this section and continue with the next

one, which is the main contribution of this paper. We hope, however, that this section might make the curious reader to start thinking about what new - in relation to other proposals - in answering the open problems in elementary particle physics and cosmology does this theory bring and, in particular with respect to this paper, the reader might start to think about the differences in the hadronic properties of the very heavy fifth family hadrons as compared to the lowest three families hadrons. Due to very strongly bound states the fifth family nuclear force differs strongly from the ordinary nuclear force.

The *Spin-Charge-Family* theory proposes in $d = (1 + (d - 1))$ ($d = 14$) dimensions a very simple action for spinors ψ - \mathcal{L}_f - which carry two kinds of the spin generators (γ^a and $\tilde{\gamma}^a$ operators), and for the corresponding gauge fields (vielbeins f^α_a and the two kinds of the spin connection fields $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, respectively) - \mathcal{L}_g - which is linear in the curvatures R and \tilde{R} .

$$S = \int d^d x E \mathcal{L}_f + \int d^d x E \mathcal{L}_g, \quad (1)$$

$$\begin{aligned} \mathcal{L}_f &= \frac{1}{2}(\bar{\psi} \gamma^a p_{0a} \psi) + h.c. \\ p_{0a} &= f^\alpha_a p_{0\alpha}, \quad p_{0\alpha} = p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}, \end{aligned} \quad (2)$$

$$\begin{aligned} \mathcal{L}_g &= (\alpha R + \tilde{\alpha} \tilde{R}), \\ R &= f^{\alpha[a} f^{\beta b]} (\omega_{ab\alpha,\beta} - \omega_{ca\alpha} \omega^c_{b\beta}), \\ \tilde{R} &= f^{\alpha[a} f^{\beta b]} (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^c_{b\beta}). \end{aligned} \quad (3)$$

Spinors carry in $d = (1 + 13)$ dimensions two kinds of the spin represented by the two kinds of the Clifford algebra objects [1-8, 12]

$$\begin{aligned} \{\gamma^a, \gamma^b\}_+ &= 2\eta^{ab} = \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+, \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0, \\ S^{ab} &= \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a), \quad \tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a), \quad \{S^{ab}, \tilde{S}^{cd}\}_- = 0, \end{aligned} \quad (4)$$

and interact correspondingly with the vielbeins and the two kinds of the spin connection fields. (f^α_a are inverted vielbeins to e^a_α with the properties $e^a_\alpha f^\alpha_b = \delta^a_b$, $e^a_\alpha f^\beta_a = \delta^\beta_\alpha$, $E = \det(e^a_\alpha)$. Latin indices $a, b, \dots, m, n, \dots, s, t, \dots$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, \dots, \mu, \nu, \dots, \sigma, \tau, \dots$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index (a, b, c, \dots and $\alpha, \beta, \gamma, \dots$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ (m, n, \dots and μ, ν, \dots), indices from the bottom of the alphabets indicate the compactified dimensions (s, t, \dots and σ, τ, \dots). We assume the signature $\eta^{ab} = \text{diag}\{1, -1, -1, \dots, -1\}$. $f^{\alpha[a} f^{\beta b]} = f^{\alpha a} f^{\beta b} - f^{\alpha b} f^{\beta a}$.

One kind of the spin, the Dirac γ^a one, explains the spin ($a = (0, 1, 2, 3)$) and all the charges ($a = (5, \dots, 14)$) [26], the other - $\tilde{\gamma}^a$ - explains the appearance of families. We kindly ask the reader to learn about all the details of the above action in Refs. [6, 7] (arXiv:1412.5866), where also the breaks of the starting symmetry, which makes the action (1) to manifest in $d = (1+3)$ effectively as the *Standard Model* action before the electroweak break, with the right handed neutrinos included, are presented.

The theory explains, why the charges of the left handed quarks and leptons (neutrinos are the regular members of each family) differ from the charges of the right handed ones. One spinor representation of $SO(1, 13)$ contains, namely, the *Standard Model* quarks and leptons: The left handed members are weak ($SU(2)_I$) charged and $SU(2)_{II}$ (this charge determines the hyper charge) chargeless, while the right handed members are weak ($SU(2)_I$) chargeless and $SU(2)_{II}$ charged. Quarks carry the colour ($SU(3) \subset$ of $SO(6)$) charge, leptons are colour chargeless. (See table IV and table III in Ref. [5], respectively.)

The theory explains also the appearance of families, the generators (\tilde{S}^{ab}) of which originate in $\widetilde{SO}(1, 13)$. (See table V and table IV in Ref. [5], respectively.) The theory predicts an even number of families, indeed two decoupled groups of four families, manifesting $\widetilde{SU}(2)_{ISO(1,3)} \times \widetilde{SU}(2)_{ISO(4)}$ and $\widetilde{SU}(2)_{II\,SO(1,3)} \times \widetilde{SU}(2)_{II\,SO(4)}$, respectively, where $(\widetilde{SO}(1, 3), \widetilde{SO}(4) \subset \widetilde{SO}(1, 7) \subset \widetilde{SO}(1, 13))$. The fourth of the lowest four families will be measured at the LHC, the lowest of the upper four families are predicted to form the dark matter [9].

Let us add that there is no evidence so far for the existence of the members of the fourth family. Calculations [20], all model dependent, in correlation with the measurements, predict that the fourth family quark masses can not be smaller than 700 GeV. The strongest limits come from the mesons decay [21], which are, assuming the existence of only one scalar field, again model dependent.

One of the authors of this paper (N.S.M.B.) (together with the coauthor G. Bregar) analyses the properties of the lower four families as predicted by the *Spin-Charge-Family* theory in Ref. [11]. In this theory several scalar fields [11] determine masses of fermions and weak bosons, replacing the Higgs's scalar and Yukawa couplings of the *Standard Model*.

One can rewrite the fermion Lagrange density \mathcal{L}_f so that a part of it (the first term in Eq. (5)) manifests the dynamical part of the *Standard Model* action for massless fermions, a part of it (the second term in Eq. (5)) represents the interaction of fermions with the scalar gauge fields which determine, after they gain nonzero vacuum expectation values and cause the electroweak break, mass matrices of the two groups of four families of fermions, while a part (the third term in Eq. (5))

causes transitions of antileptons into quarks and antiquarks into quarks (and back)

$$\begin{aligned}
\mathcal{L}_f &= \bar{\psi} \gamma^m (p_m - \sum_{A,i} g^A \tau^{Ai} A_m^{Ai}) \psi + \\
&\quad \{ \sum_{s=7,8} \bar{\psi} \gamma^s p_{0s} \psi \} + \\
&\quad \{ \sum_{t=5,6,9,\dots,14} \bar{\psi} \gamma^t p_{0t} \psi \}, \\
p_{0s} &= p_s - \frac{1}{2} S^{s's''} \omega_{s's''s} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{abs}, \\
p_{0t} &= p_t - \frac{1}{2} S^{t't''} \omega_{t't''t} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{abt}.
\end{aligned} \tag{5}$$

Here $m \in (0, 1, 2, 3)$, $s \in (7, 8)$, $(s', s'') \in (5, 6, 7, 8)$, (a, b) (appearing in \tilde{S}^{ab}) run within $(0, 1, 2, 3)$ and $(5, 6, 7, 8)$, $t \in (5, 6, 9, \dots, 13, 14)$, $(t', t'') \in (5, 6, 7, 8)$ and $\in (9, 10, \dots, 14)$. The spinor function ψ represents all family members of all the $2^{\frac{7+1}{2}-1}$ families, which are before the electroweak break massless due to the mass protection caused by the fact that the left and right handed members carry different weak and hyper charges (table III and table IV in Refs. [5]). The operators τ^{Ai} ($= \sum_{a,b} c^{Ai}_{ab} S^{ab}$) determine the hyper charge ($A = 1$), the weak charge ($A = 2$) and the colour charge ($A = 3$): $\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak}$, $f^{1ijk} = 0$, $f^{2ijk} = \varepsilon^{ijk}$ and f^{3ijk} is the $SU(3)$ structure tensor.

All the scalar fields from the second term in Eq. (5) carry, due to their scalar index $s = (7, 8)$, the weak charge and the hyper charge $\mp \frac{1}{2}$ and $\pm \frac{1}{2}$, respectively, as does the Higgs's scalar in the *Standard Model*. They carry additional quantum numbers in adjoint representations (the family quantum numbers originating in \widetilde{S}^{ab} and the quantum numbers Q, Q', Y' (we kindly ask the reader to look for the detailed explanation in Refs. [5–7]) and determine, after they gain nonzero vacuum expectation values, together with the vector gauge fields in loop corrections in all orders mass matrices of the two groups of four families. These scalar fields predict that at the LHC will be measured not only the fourth family quarks, but also several scalar fields [11].

The evaluation of masses and mixing matrices of the lower four families [10] suggests that the (stable) fifth family masses, which form the dark matter, should be above a few TeV, while evaluations of the breaks of symmetries from the starting one (Eq. 1) suggests that these masses should be far bellow 10^{10} TeV.

Following the history of the fifth family members in the expanding universe up to today and estimating also the scattering properties of this fifth family on the ordinary matter [9], the evaluated masses of the fifth family quarks, under the assumption that the lowest mass fifth family baryon

is the fifth family neutron, are in the interval

$$\text{a few times } 10 \text{ TeV} < m_5 < 10^5 \text{ TeV}. \quad (6)$$

In the Refs. [9] was predicted that if DAMA/LIBRA [13, 14] really measures the fifth family neutrons, also other direct experiments like CDMS [15], CREST [16] and XENON100 [17] should in a few years observe the dark matter clusters. One of the authors of this paper (S.N.M.B, together with B.M. [18]) put a lot of efforts to understand, why DAMA/LIBRA measures the dark matter while the other experiments do not (yet?) [14, 18]. In Ref. [14] one can read the answers to several questions which also the reader might have.

III. THE VERY HEAVY (FIFTH FAMILY STABLE) NEUTRON AS A CANDIDATE FOR THE DARK MATTER

In this section we study the scattering cross section of a very heavy baryon with the ordinary nucleon, provided that the ratio of masses $\frac{m_5}{m_1}$ is large enough ($\geq 10^3$). Since the dark matter constituent with the electromagnetic charge would lead to a substantial Rutherford scattering, what would not be in agreement with the observed dark matter properties, we study the baryon with zero electromagnetic charge, estimating limits on $(u_5 - d_5)$ quark mass differences under which the neutral baryon n_5 appears as the lightest baryon (Subsect. III A).

From the (locally quite approximately) known mass density of the dark matter and from the freezing out procedure of the fifth family neutrons in the evolution of the universe it is deduced in Ref. [9] (Eq. (7)) that the mass of the very heavy fifth family neutrons must be in the interval from a few ten TeV to a few 100 TeV.

The weak cross section, which appears to be independent of the heavy neutron mass (Eq. (14)), is in agreement with the direct experiments of the dark matter [13], if the our very heavy neutrons (Subsect. III B) have a mass roughly 100 TeV or larger.

In Subsect. III C we show within the one gluon exchange approximation that for a heavy neutron with the mass of ≥ 100 TeV the scattering cross section due to the colour interaction is much smaller than the cross section due to the weak interaction.

Let us, before starting with the formal evaluations, try to understand what is happening when a weak charged colourless cluster of very heavy and consequently strongly bound quarks, with the electromagnetic charge equal to zero, approaches nucleus of the ordinary (first family weak charged) baryons. Let their relative velocity be small so that the kinetic energy of the lighter cluster with

respect to their center of mass motion (that is almost with respect to the heavier cluster) is small in comparison with the binding energy of nucleons in the nucleus. The ordinary nucleus interact with the heavy quark cluster through the weak and through the colour force.

The size of any ordinary quark in the nucleus is huge in comparison with our very heavy and very tiny fifth family neutron. The very quark, which "hits" our heavy neutron through the weak and the colour force, reflects elastically from our very heavy neutron. However, since it is strongly bound into the ordinary baryon, while baryon is strongly bound into the nucleus, the whole nucleus elastically scatter from our heavy neutron (almost) as from the rigid wall, while our neutron continues its way with almost unchanged direction and velocity.

It is not difficult to calculate the contribution of the weak force to the scattering amplitude for such an event. This is done in Subsect. III B. The evaluation of the contribution of the colour force needs more effort. In Ref. [9] it was assumed that within two orders of magnitude this scattering amplitude is proportional to the size of the tiny heavy neutron. In scattering of ordinary neutrons on ordinary nucleons the contribution of the nuclear (that is of the colour) force is in the low energy regime indeed proportional to the size of the nucleons. But in ordinary nucleon the quarks are "dressed" into the large gluon cloud which determine the quark mass in the nucleon, while in the heavy neutron case one gluon exchange dominates in the binding energy.

In the present study we take into account that the gluons can "see" the tiny heavy neutron only if the heavy neutron polarizes during the scattering. We calculate dipole-dipole scattering only. This is done in Subsect. III C.

A. Is the very heavy neutron the lightest fifth family baryon?

First we calculate the dominant properties of a three-quark cluster, its binding energy and size (more details can be found in Ref. [9]). For this purpose we assume equal masses of all the fifth family quarks and we realize that in the regime that $m_{n_5} >$ than a few TeV (table I in Ref. [9]) the colour interaction is coulombic - one gluon exchange dominates - and so is also the weak and the electromagnetic interaction, all determined by one massless weak boson and one photon exchange, respectively.

For three nonrelativistic particles with attractive coulombic like interaction we solve the equations of motion for the Hamiltonian [9]

$$H = 3m_5 + \sum_i \frac{\vec{p}_i^2}{2m_5} - \frac{(\sum_i \vec{p}_i)^2}{6m_5} - \sum_{i < j} \frac{2}{3} \frac{\alpha_s}{r_{ij}}. \quad (7)$$

The potential energy of the solution can be parameterized as

$$V_s = -\frac{2}{3}\alpha_s\epsilon, \quad \epsilon = \left\langle \sum_{i<j} \frac{1}{r_{ij}} \right\rangle = 3\eta\alpha_s m_5, \quad (8)$$

where m_5 is the average mass of quarks in the fifth family. The parameter η for a variational solution using Jacobi coordinates and exponential profiles was calculated in Ref. [9]: $\eta = 0.66$. The binding energy is then

$$E = \frac{1}{2} V_s = -E_{\text{kin}} = -\alpha_s^2 \eta m_5. \quad (9)$$

The splitting of baryons in the fifth family is caused by the $(u_5 - d_5)$ mass difference (due to the interaction with the scalar fields carrying the weak, the hyper charge and the family charges interactions), as well as by the potential energy of the electromagnetic and weak interaction. Even if we are far above the electroweak phase transition we can still work in the basis using Weinberg mixing of γ and Z , which is familiar to the low energy hadron physicists. We shall therefore use this basis.

Let us split the electro-weak interaction into five contributions: i. electric, ii. Z-exchange Fermi (vector), iii. Z-exchange Gamov-Teller (axial), iv. W-exchange Fermi (vector) and v. W-exchange Gamov-Teller (axial) contributions. We obtain for the mass M of the heavy cluster the expression

$$M = \sum_i m_{5i} + E + (V_{\text{EM}} + V_Z^{\text{F}} + V_Z^{\text{GT}} + V_W^{\text{F}} + V_W^{\text{GT}}), \quad (10)$$

where the corresponding five contributions are as follows

$$\begin{aligned} V_{\text{EM}} &= \left\langle \sum_{i<j} q_{5i} q_{5j} \right\rangle \alpha_{\text{EM}} \epsilon, \\ V_Z^{\text{F}} &= \left\langle \sum_{i<j} \left(\frac{t_i^3}{2} - \sin^2 \vartheta_W q_{5i} \right) \left(\frac{t_j^3}{2} - \sin^2 \vartheta_W q_{5j} \right) \right\rangle \alpha_Z \epsilon, \\ V_Z^{\text{GT}} &= \left\langle \sum_{i<j} \frac{t_i^3 t_j^3}{4} \vec{\sigma}_i \vec{\sigma}_j \right\rangle \alpha_Z \epsilon, \\ V_W^{\text{F}} &= \left\langle \sum_{i<j} \frac{t_i^- t_j^+ + t_i^+ t_j^-}{8} \right\rangle \alpha_W \epsilon, \\ V_W^{\text{GT}} &= \left\langle \sum_{i<j} \frac{t_i^- t_j^+ + t_i^+ t_j^-}{8} \vec{\sigma}_i \vec{\sigma}_j \right\rangle \alpha_W \epsilon. \end{aligned} \quad (11)$$

The operators $\vec{t} = \frac{1}{2}\vec{\tau}$ are the isospin operators, $t^\pm = (t_1 \pm it_2)$, and $\vec{\sigma}$ are the Pauli spin matrices, q_{5i} are the electromagnetic charges of quarks.

The numerical values for all these five terms are, for a particular choice of the parameters, presented in table I, where we choose for the average quark mass $m_5 = 100$ TeV with the corresponding average momentum of each quark $p = \sqrt{2m_5 E_{\text{kin}}/3} = 5.1$ TeV. At this momentum scale, we read the running coupling constants from Particle Data Group diagram [25] as $\alpha_s = \alpha_3 = 1/13$, $\alpha_W = \alpha_2 = 1/32$ and $\alpha_1 = 1/56$. It then follows that $\sin^2 \vartheta_W = (1 + \frac{5}{3} \frac{\alpha_W}{\alpha_1})^{-1} = 0.255 \approx 1/4$, $\alpha_{\text{EM}} = \alpha_W \sin^2 \vartheta_W = 1/128$ and $\alpha_Z = \alpha_W / \cos^2 \vartheta_W = 1/24$.

	$u_5 u_5 u_5$	$u_5 u_5 d_5$	$u_5 d_5 d_5$	$d_5 d_5 d_5$
$V_{\text{EM}}/\epsilon\alpha_{\text{EM}}$	+4/3	0	-1/3	+1/3
$V_Z^{\text{F}}/\epsilon\alpha_Z$	+1/48	-1/48	0	+4/48
$V_Z^{\text{GT}}(n_5)/\epsilon\alpha_Z$	—	-15/48	-15/48	—
$V_Z^{\text{GT}}(\Delta_5)/\epsilon\alpha_Z$	-9/48	+3/48	+3/48	-9/48
$V_W^{\text{F}}/\epsilon\alpha_W$	0	+1/4	+1/4	0
$V_W^{\text{GT}}(n_5)/\epsilon\alpha_W$	—	-30/48	-30/48	—
$V_W^{\text{GT}}(\Delta_5)/\epsilon\alpha_W$	0	-1/4	-1/4	0
$V_{\text{EW}}(n_5)/\epsilon$	—	-0.0256	-0.0273	—
$V_{\text{EW}}(\Delta_5)/\epsilon$	+0.0035	+0.0017	-0.0000	-0.0017

TABLE I: Electro-weak contributions to the fifth family - very heavy- baryon masses for the five terms in Eq. (11). The values for the parameters are presented in the text. The lowest two lines are the sum over above contributions. The unnecessary decimal places are there if the reader likes to check the reproducibility of the results, — means non applicable. One notices that the vector contributions are the same for neutrons (n_5) and deltas (Δ_5) baryons, while the axial contributions differ dramatically.

In this example, the binding energy $E = -0.39$ TeV and the average reciprocal distance $\langle 1/r_{ij} \rangle = \epsilon/3 = \eta\alpha_s m_5 = 5.1$ TeV = $2.6 \cdot 10^4 \text{ fm}^{-1}$.

One finds for the limits on $(u_5 - d_5)$ mass difference, which make the neutral baryon n_5 the lightest, the following relations

$$\begin{aligned}
 (m_{u5} - m_{d5}) &< (0.0273 - 0.0017) \epsilon = 0.0256 \epsilon, \quad \text{preventing } (u_5 d_5 d_5) \rightarrow (d_5 d_5 d_5), \\
 (m_{u5} - m_{d5}) &> (-0.0273 + 0.0256) \epsilon = -0.0017 \epsilon, \quad \text{preventing } (u_5 d_5 d_5) \rightarrow (u_5 u_5 d_5). \quad (12)
 \end{aligned}$$

For our value of $\epsilon = 15.24$ TeV the above requirements lead to the relation

$$-0.026 \text{ TeV} < (m_{u5} - m_{d5}) < 0.39 \text{ TeV}. \quad (13)$$

This limits are narrow compared to the mass scale $m_5 = 100$ TeV, but they are not so narrow if

the mass generating mechanism would be of the order of 100 GeV as it is in the case of the so far observable families.

B. The weak $(u_5 d_5 d_5) - (u_1 d_1 d_1)$ cross section

It is easy to calculate the weak scattering amplitude for $(u_5 d_5 d_5) - (u_1 d_1 d_1)$, since the very heavy neutron (n_5) is a point particle compared to the range of the weak interaction and its quark structure is negligible. Only Z-exchange part matters since there is not enough energy to excite $(u_5 d_5 d_5)$ into $(d_5 d_5 d_5)$ or $(u_5 u_5 d_5)$ via W-exchange. We consider only the scattering on neutron (the "charge" of a proton (almost) happens to cancel). Also, we consider only the Fermi (vector) matrix element, since it contributes coherently in heavy nuclei, while the Gamov-Teller (axial) has many cancellations in spin coupling. In Eq. (14) the matrix element and the scattering amplitude are presented.

$$\begin{aligned}\mathcal{M} &= \left[\frac{1}{2}t_1^3 - \sin^2 \vartheta_W q_1\right] \frac{g_Z^2}{m_Z^2} \left[\frac{1}{2}t_5^3 - \sin^2 \vartheta_W q_5\right] = \frac{G_F}{2\sqrt{2}}, \\ \sigma_n &= 2\pi |\mathcal{M}|^2 \frac{4\pi p_1^2}{(2\pi)^3 v^2} = \frac{m_{n_1}^2}{\pi} |\mathcal{M}|^2 = \frac{G_F^2 m_{n_1}^2}{8\pi} = 1.9 \times 10^{-13} \text{fm}^2,\end{aligned}\quad (14)$$

Index $_1$ refers to the first family fermions, $_5$ to the fifth family ones. We note that the cross section does not depend on the mass m_{n_5} provided it is much larger than m_{n_1} (the first family nucleon mass). For the scattering amplitude of our n_5 on a target with A nucleons and Z protons we have $\sigma_A = \sigma_n (A - Z)^2 A^2$, since at low enough velocity ($100 < v < 300$) km/s of the heavy neutron, n_5 , scattering is coherent [9].

Let us now evaluate the rate at a detector of $^{23}_{11}\text{Na}$ $^{127}_{53}\text{I}$ per kilogram of detector

$$\begin{aligned}R_{1kg} &= \sigma_A N_A \frac{\rho_{n_5} v}{m_{n_5}} \\ R_{1kg} &= \sigma_n [(A_{Na} - Z_{Na})^2 A_{Na}^2 + (A_I - Z_I)^2 A_I^2] \frac{N_A v}{A_{Na} + A_I} \frac{\rho_{n_5}}{m_{n_5}} = 1.3/\text{day},\end{aligned}\quad (15)$$

where Av is the Avogadro number, and where $\rho_{n_5} = 0.3 \text{ GeV cm}^{-3}$, $m_{n_5} = 300 \text{ TeV}$ (for $m_{q_5} = 100 \text{ TeV}$), $v = 230 \text{ km/s}$. More details can be found in Ref. [9]. (The bound is only approximate since $(100 < v < 300) \text{ km/s}$.)

This is in agreement with the rate claimed by the DAMA/LIBRA [13] collaboration: $\Delta R_{1kg}(\text{DAMA}) = 0.02/\text{day}$, $R_{1kg}(\text{DAMA}) \sim (0.1 \leftrightarrow 1)/\text{day}$.

Should the DAMA results turn out to be smaller, or would not be confirmed, then m_{q_5} should be even larger, if for the velocity of Sun $v = 230 \text{ km/s}$ is an acceptable value [9].

C. The colour heavy meson – light meson cross section

We estimate the scattering cross section of $(u_5 d_5 d_5)$ on $(u_1 d_1 d_1)$, or on any first family nucleon, in one boson exchange approximation. A detailed calculation is a demanding few body problem even in the lowest order approximation. We get the answer to the question to which extent the assumption that this cross section is within a factor of a few 10 proportional to the square of the size of the fifth family nucleon (n_5) , as it is assumed in Ref. [9], by calculating the simpler problem: the meson₅ - meson₁ scattering within one boson exchange approximation. In this case we trust that the baryon in a quark-diquark approximation resembles a meson.

Let us point out that the evaluation, presented in this section, is very relevant also for bottomium or future heavy baryons in the $(10 - 100)$ GeV region scattering on the first family baryons.

In the equation below (16) we present the trial functions of the light and the heavy meson, together with relevant quantities, such as the chromomagnetic dipole moment D (in our case D_5 of the heavy meson sitting in the dipole field G_1 of the light meson. We use m_1 and m_5 for the light and heavy quark masses, respectively, and $q_1, \bar{q}_1, q_i, \bar{q}_i, i = (1, 2)$ represent quark and antiquark for either light ($i = 1$) or heavy ($i = 5$) quarks. The interaction strength is $\alpha = \frac{4}{3}\alpha_s$. \vec{r} and \vec{R} are relative coordinates of light and heavy mesons. The parameters b and B are the widths of the hydrogen-like (actually positronium-like) wave functions for the light or heavy quark-antiquark pair. In the virtual excited state of the light meson, the width f is determined by minimizing the energy of the composite light meson – heavy meson system. In the virtual excited state of the heavy meson we keep the same width as in the ground state since the change due to minimization was negligible. This is expected since the light system can adapt easily due to small excitation energies while the heavy system is too “expensive”.

$$\begin{aligned}
\vec{r} &= \vec{r}_{q_1} - \vec{r}_{\bar{q}_1}, & b &= 1/(\frac{1}{2}m_1)\alpha & \vec{R} &= \vec{R}_{q_5} - \vec{R}_{\bar{q}_5}, & B &= 1/(\frac{1}{2}m_5)\alpha \ll b \\
\psi_0 &= (2/\sqrt{4\pi b^3}) \exp(-r/b) & & & \Psi_0 &= (2/\sqrt{4\pi B^3}) \exp(-R/B) \\
\psi_z &= \frac{2^{-3/2}}{\sqrt{4\pi f^3}} (r/f) \cos \vartheta \exp(-r/f) & & & \Psi_z &= \frac{2^{-3/2}}{\sqrt{4\pi B^3}} (R/B) \cos \Theta \exp(-R/B) \\
\epsilon_0 &= -(1/2)(\frac{1}{2}m_1)\alpha^2 & & & E_0 &= -(1/2)(\frac{1}{2}m_5)\alpha^2 \\
\epsilon_{z,kin} &= +(1/8)(\frac{1}{2}m_1)\alpha^2 (b/f)^2 & & & E_z &= -(1/8)(\frac{1}{2}m_5)\alpha^2 \\
G_{z_1} &= \langle \psi_z | z / (r/2)^3 | \psi_0 \rangle = \gamma / \sqrt{fb^3} & & & D_5 &= \langle \Psi_z | z_5 | \Psi_0 \rangle = \beta B
\end{aligned} \tag{16}$$

The numerical factors are denoted as $\gamma = 16\sqrt{2}/3 = 7.542$ and $\beta = 2^{15/2}/3^5 = 0.745$.

The meson wave functions carry also a factor corresponding to the colour degree of freedom. The

three single particle colour eigenstates (“red”, “green” and “blue”) are denoted as $\mathcal{R} = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$, $\mathcal{B} = (-\frac{1}{2}, \frac{1}{2\sqrt{3}})$, $\mathcal{G} = (0, -\frac{1}{\sqrt{3}})$ while the anticolour states are $\bar{\mathcal{R}} = (-\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, $\bar{\mathcal{B}} = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, $\bar{\mathcal{G}} = (0, \frac{1}{\sqrt{3}})$.

The whole states, for the spatial and the colour part, are then for the light meson

$$\phi_0 = \psi_0 \frac{(|\mathcal{R}\rangle |\bar{\mathcal{R}}\rangle + |\mathcal{G}\rangle |\bar{\mathcal{G}}\rangle + |\mathcal{B}\rangle |\bar{\mathcal{B}}\rangle)}{\sqrt{3}}, \quad \phi_{z3} = \psi_z \frac{(|\mathcal{R}\rangle |\bar{\mathcal{R}}\rangle - |\mathcal{G}\rangle |\bar{\mathcal{G}}\rangle)}{\sqrt{2}},$$

and for the heavy meson

$$\Phi_0 = \Psi_0 \frac{(|\mathcal{R}\rangle |\bar{\mathcal{R}}\rangle + |\mathcal{G}\rangle |\bar{\mathcal{G}}\rangle + |\mathcal{B}\rangle |\bar{\mathcal{B}}\rangle)}{\sqrt{3}}, \quad \Phi_{z3} = \Psi_z \frac{(|\mathcal{R}\rangle |\bar{\mathcal{R}}\rangle - |\mathcal{G}\rangle |\bar{\mathcal{G}}\rangle)}{\sqrt{2}}.$$

Only the spatial excitation in the z -direction and the excitation corresponding to the colour operator λ_q^3 are in Eq. (16) written explicitly. Other directions and other operators can be written in a similar way.

We need the colour matrix element

$$\left\langle \frac{(|\mathcal{R}\rangle |\bar{\mathcal{R}}\rangle - |\mathcal{G}\rangle |\bar{\mathcal{G}}\rangle)}{\sqrt{2}} \left| \frac{\lambda_{q1}^3}{2} - \frac{\lambda_{\bar{q}1}^3}{2} \right| \frac{(|\mathcal{R}\rangle |\bar{\mathcal{R}}\rangle + |\mathcal{G}\rangle |\bar{\mathcal{G}}\rangle + |\mathcal{B}\rangle |\bar{\mathcal{B}}\rangle)}{\sqrt{3}} \right\rangle = \sqrt{\frac{2}{3}}$$

For the colour neutral hadrons, the dominant term in the expansion is the effective dipole–dipole, colour-octet – colour-octet potential

$$\hat{V}_{\text{dipole}} = \alpha_s \left(\vec{R}_{q5} \frac{\vec{\lambda}_{q5}}{2} + \vec{R}_{\bar{q}5} \frac{\vec{\lambda}_{\bar{q}5}}{2} \right) \cdot \left(\frac{\vec{r}_{q1}}{r_{q5}^3} \frac{\vec{\lambda}_{q1}}{2} + \frac{\vec{r}_{\bar{q}1}}{r_{\bar{q}1}^3} \frac{\vec{\lambda}_{\bar{q}1}}{2} \right).$$

There are two scalar products, between \vec{R} and \vec{r} and between $\vec{\lambda}$ and $\vec{\lambda}$. The perturbation term between the unperturbed ground state and the virtual excitation is then

$$V_{z,3} = \alpha_s \langle \Psi_z \psi_z | \{ \frac{R_3}{2} \} \sqrt{\frac{2}{3}} \{ \frac{r_3/2}{(r/2)^3} \} \sqrt{\frac{2}{3}} | \Psi_0 \psi_0 \rangle = \frac{\alpha_s D_{5r3} G_{13}}{6}$$

$$V_{r1,\omega} = V_{r2,\omega} = V_{r3,\omega} \equiv V \quad \text{equal for all } \omega.$$

where the index $\omega = 1, \dots, 8$ corresponds to all colour excitations.

The second order perturbation theory then gives the effective potential between the two clusters

$$V_{eff} = -24 \frac{V^2}{(E_z - E_0) + \epsilon_{z,kin}}.$$

We neglected $\epsilon_{z,pot}$ and ϵ_0 . The factor 24 comes from 3 spatial and 8 colour degrees of freedom.

$$V_{eff} = -\frac{2}{3}, \frac{(\alpha_s D_{5z} G_{1z})^2}{(3/8)(\frac{1}{2}m_5)(4\alpha_s/3)^2 + (1/8)(\frac{1}{2}m_1)(4\alpha_s/3)^2(b/f)^2}$$

$$V_{eff} = -\frac{2(\beta\gamma B)^2}{fb^3(m_5 + (1/3)m_1(b/f)^2)}$$

Note that α_s has cancelled. Minimization

with respect to f gives $f/b = \sqrt{m/3M} \ll 1$. Finally, we get

$$V_{eff} = -\frac{\sqrt{3}\beta^2\gamma^2}{b^3} \left(\frac{m_1}{m_5}\right)^{3/2} \frac{B}{m_1}$$

Here we take that the distance between the two clusters is equal to zero, $U = 0$. We assume

$$V_{eff}(U) = V_{eff}(U = 0) \exp(-2U/b).$$

In Born approximation (with the mass of the lighter cluster $m_{q_1} + m_{\bar{q}_1} = 2m_1$) we get

$$a = \frac{(2m_1)}{2\pi} \int V_{eff}(U) d^3U = \sqrt{3}\beta^2\gamma^2 \left(\frac{m_1}{m_5}\right)^{3/2} B. \quad (17)$$

This is our main result, that is the scattering amplitude in the Born approximation, which is obviously not just proportional to the size of the meson. In order to see how large is this value in comparison with the size of the heavy meson, which is of the order of the width $B = 1/(\frac{1}{2}m_5)\alpha \approx 4 \cdot 10^{-5}$ fm let us now give a numerical example with the choice

$$m_1 = 300 \text{ MeV}, \quad m_5 = 100 \text{ TeV}, \quad m_1/m_5 = 3 \cdot 10^{-6}, \quad \alpha_s = 1/13$$

$$a = \sqrt{3}\beta^2\gamma^2 \left(\frac{m_1}{m_5}\right)^{3/2} B = 1.1 \cdot 10^{-11} \text{ fm}. \quad (18)$$

The cross section corresponding to this scattering amplitude is

$$\sigma = 4\pi a^2 = 1.5 \cdot 10^{-21} \text{ fm}^2. \quad (19)$$

IV. CONCLUSION

In this paper we discuss scattering amplitudes of very heavy stable electromagnetically neutral neutrons (n_5) on the first family neutrons (n_1) within the one boson exchange approximation. The fifth family stable very heavy neutrons are, namely, predicted by the *Spin-Charge-family* theory, proposed by one of the authors [1–7, 12] and are candidates for the dark matter.

The *Spin-Charge-family* theory is very promising to be the right step beyond the *Standard Model*: It offers an explanation not only for the assumptions of the *Standard Model*, with the appearance of families included, but also for other phenomenological facts, such as the dark matter

and the matter-antimatter asymmetry in our universe. We hope that the reader will enjoy this theory and would hopefully like to contribute to its application.

The evaluations of the scattering amplitudes for the stable heavy (fifth family) neutron is presented in Sect. III. The amplitudes are calculated in the one boson exchange approximation, replacing the heavy and the first family neutrons (n_5, n_1) with the simpler meson₅ and meson₁ to check whether a rough estimation [9], that the scattering amplitude caused by the colour force is proportional to the size of n_5 , is an acceptable approximation for very heavy neutrons. It turned out that the scattering amplitude at least in the colour dipole - colour dipole approximation is much smaller than the one estimated roughly in Ref. [9].

This paper is written also in purpose to convince the hadron physicists that if the *Spin-Charge-Family* theory is the right step beyond the *Standard Model* then they will have a pleasant time to study properties of the clusters forming dark matter with their knowledge from the lower three families. It is interesting to notice how much does the "nuclear" physics change when the constituents are very heavy. We demonstrate in Sect. III that for the mass of the fifth family neutron (n_5) equal or greater than 100 TeV, the weak force is stronger than the "fifth family-first family" nuclear force.

We also offer the present calculations as an introduction to study of heavy hadron – light hadron scattering.

Although either meson-meson or the one boson exchange approximations, used in our calculation, might not be accurately enough yet the extremely small ratio between the weak and the "fifth family-first family" nuclear force in the case that $m_{n_5} \approx 100$ TeV or larger tells that the nuclear force among very heavy objects is very very weak.

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- [26] The operators γ^a are matrices after we make a choice of the basis and find their representations. One can find in Refs. [6, 7] a short explanation for such a choice of basis. There also the expressions for all the operators - those which operates on spins degrees of freedom and those which operates on charge or family quantum numbers - can be found.